Test of the Unitary Pole Approximation to the Malfliet-Tjon Potential for Elastic Neutron-Deuteron Scattering *

H. Ziegelmann

Institut für Theoretische Physik der Universität Tübingen,
Tübingen, Germany

The quality of the unitary pole approximation to the soft-core Malfliet-Tjon potential is demonstrated for elastic neutron-deuteron scattering above breakup threshold.

Recently Alt and Bakker [1] have shown that the unitary pole approximation (UPA) [2] to the soft-core Malfliet-Tjon potential [3] gives very good results in the three-nucleon system below threshold. According to Ref. 1 it is now possible to calculate three-nucleon observables for local soft-core potentials with less effort than was previously necessary. Instead of solving the two-dimensional Faddeev equations [3,4] for the local potential, it is sufficient now to solve the one-dimensional Amado-Lovelace equations for the corresponding rank-one separable potential obtained by the UPA.

The success of the UPA can be explained in the following way: the form factors of the UPA, which are constructed with the help of the eigenfunctions belonging to the largest attractive eigenvalue of the two-particle Lippmann-Schwinger kernel, have a node [2]. Therefore the separable potential is partly attractive and partly repulsive, in contrast to the commonly used Yamaguchi potential which has too much attraction. The success of the UPA has been explained in a more sophisticated way by Bakker and Sandhas using the quasiparticle method [5]. This investigation also shows how UPA calculations can be improved (if necessary) and it was in fact possible to reproduce the results of Ref. [3,4] below threshold by calculating first order corrections [5,6].

It is the aim of the present investigation to check the quality of the UPA for positive c.m. energies in order to show that the treatment of a three-body system interacting via local soft-core potentials can also be simplified in this energy region. Using the real axis method of Ref. [7] we solved the singular Amado-Lovelace equations for the UPA. The real-
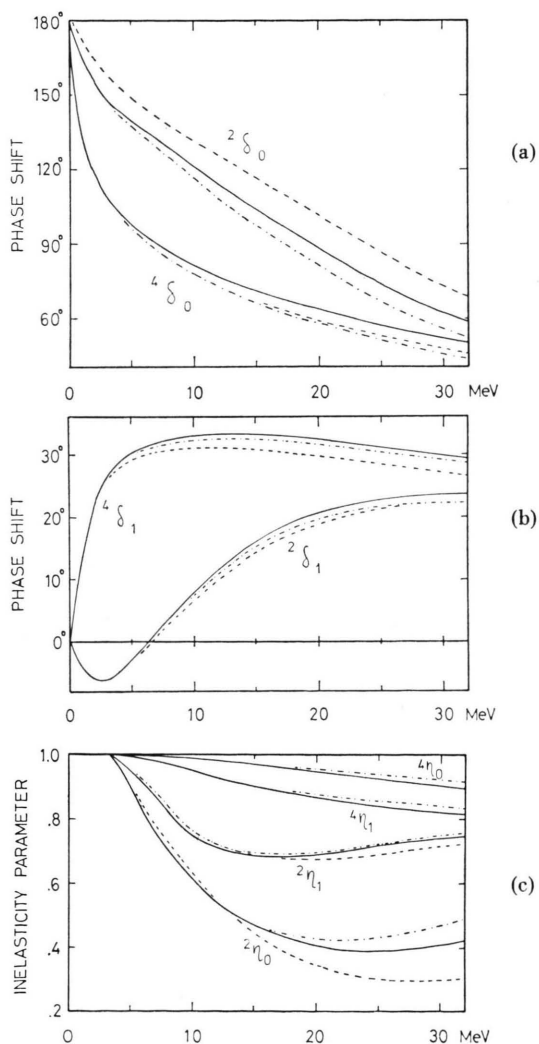
Fig. 1. Real part of n—d phaseshifts and inelasticity parameter versus neutron laboratory energy. The full lines are the exact results of Reference 4. The dashed-dotted lines are the UPA results. The dashed lines are the results for a Yamaguchi potential.

part of the calculated phaseshifts is shown in Fig. 1(a, b), the inelasticity parameter in Fig. 1(c) in comparison with the exact results of Kloet and Tjon [4]. We also show results for the Yamaguchi potential T5/S5 introduced by Bakker [2], which gives approximatively the same low energy data in the two-nucleon system as the Malfliet-Tjon potential. The three-body results for this Yamaguchi potential are closer to the exact results of Kloet and Tjon than the results of Sloan [8], which have been used in Ref. [4] for comparison. Nevertheless the agreement between the UPA and the exact calculations is much better than for the Yamaguchi potential, except for the quartet S- and the doublet P-wave at higher energies. Where no results for the Yamaguchi potential are shown, they coincide with the UPA. Higher partial waves are not sensitive to potential details, and here Malfliet-Tjon potential, UPA and Yamaguchi potential give almost identical results.
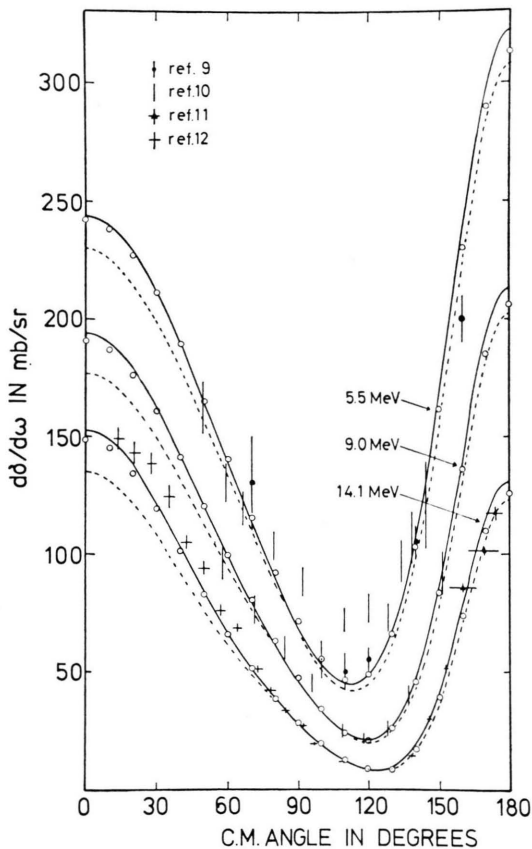
For the calculation of differential cross sections it is important that the UPA gives a quartet P-wave phaseshift, which is very close to the exact one for all energies. As Fig. 2 shows, the UPA cross sections are very close to the exact ones. In forward direction we observe almost the same improvement over the Yamaguchi results as Kloet and Tjon [4]. For higher energies (see Fig. 3) the UPA is still better
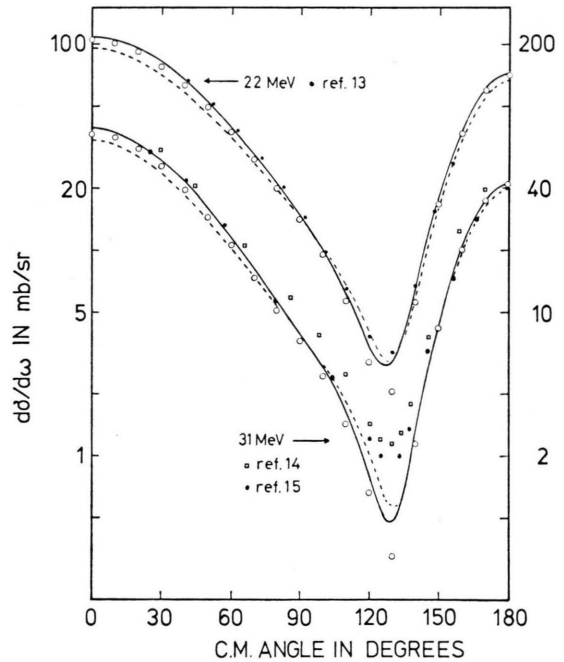

Fig. 3. The same as Figure 2. Experimental data are taken from p−d experiments. Left hand scale refers to 22 MeV, right hand scale to 31 MeV.

Fig. 2. Experimental and theoretical differential cross section for n−d scattering at 5.5, 9, 14.1 MeV. The full line is the exact result of Reference 4. The dashed line is the result for a Yamaguchi potential. The UPA result is shown by open circles.

than the Yamaguchi result in forward direction, but it fails to reproduce the minimum at 130°. Here the Yamaguchi result is even closer to the experimental data than the results of Kloet and Tjon. Nevertheless one cannot say that the Yamaguchi potential is more realistic in this case than the Malfliet-Tjon potential, because the high value of the cross section in the minimum can be directly related to the missing repulsion in the Yamaguchi potential: the missing repulsion gives an overbinding in the triton, a negative n-d scattering length and a too high value for the doublet S-wave phaseshift [see Fig. 1 a)]. As the quartet contribution almost vanishes, this high value of the phaseshift shows up in a high value of the cross section minimum. It is felt that the high experimental value of the minimum cannot be explained by three-body models with only central

forces (in order to fit the minimum v. Oers and Brockmann [16] had to assume $^2\delta_0 = 125°$ $^2\eta_0 = 0.66$!).

Table 1. Total elastic and total breakup cross section.

Energy (MeV, Lab)	5.5	9.	14.1	22.	31.
Total elastic cross section (mb)					
Exact [4]	1465	970	624	365	221
UPA	1467	961	608	346	203
Yamaguchi	1393	911	574	331	199
Total breakup cross section (mb)					
Exact [4]	69	145	164	150	127
UPA	63	137	158	144	120
Yamaguchi	59	135	159	148	125

Comparing the total elastic cross sections (Table I) we see that the UPA is better than the Yamaguchi approximation except at higher energies, where in the Yamaguchi result the too low forward peak is almost cancelled by the higher minimum at 130°. For the total breakup cross section the Yamaguchi approximation seems to be superior to the UPA at higher energies. The reason for this is again a cancellation effect. Looking at the inelasticity parameters [Fig. 1 c)] we see that the UPA gives systematically too high values of η and therefore too low values for the total breakup cross section. In contrast to this the Yamaguchi results lie at higher energies on different sides of the exact curve which results in a too low quartet contribution and a too

high doublet contribution. Therefore the total breakup cross section for the Yamaguchi potential is closer to the exact result than the UPA result, although the UPA results in each partial wave are as accurate or sometimes much more accurate than the Yamaguchi results.

We therefore conclude that the UPA is a very good approximation to the Malfliet-Tjon potential up to 14 MeV. In this energy region the differential cross sections can be calculated with less effort than in Ref. [4] and the results are almost as good as the exact ones. Our calculations show that the S-wave part of a realistic nucleon-nucleon interaction can be treated in a much simpler way than was done previously, and that is just what we need, if we want to do n-d calculations with more complicated interactions. If one wants more accuracy or if one wants to go to higher energies, S- and P-wave n-d scattering should be calculated by taking into account the difference between the Malfliet-Tjon potential and the UPA in first order of perturbation theory, as was done in Ref. [5, 6].

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